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Possibility of Enhancement of Amplitude-Squared Squeezing in Mixing with Coherent Light Beam Using a Mach–Zehnder Interferometer

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Possibility of enhancement of amplitude-squared squeezing for Mach–Zehnder interferometer is investigated and it is found that the maximum amount of amplitude-squared squeezing obtained by the present mixing is not greater than that obtained by Prakash and Mishra.

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1. Introduction

Sudarshan and Glauber [1, 2], independently, showed that the density operator of radiation can be written as

$$\hat{\rho} = \int \mathrm{d}^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|, \tag{1}$$

where $\alpha \equiv \alpha_{\mathrm{r}} + \mathrm{i}\alpha_{\mathrm{i}}$ is a complex number, $\mathrm{d}^2\alpha \equiv \mathrm{d}\alpha_{\mathrm{r}}\mathrm{d}\alpha_{\mathrm{i}}$ and $|\alpha\rangle$ is the eigenstate of the annihilation operator $\hat{a}$ ($\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$), carets ($\hat{}$) denote operators, and is called a coherent state [1]. Radiation is said to be in a classical state if the weight function $P(\alpha)$ is not more singular than Dirac delta function $\delta^2(\alpha)$ or is non-negative whereas non-classical features do not follow such a condition. Earlier, the non-classical features, antibunching [3, 4] and squeezing [5–7], were studied with academic interest [3–6, 8], as they provide instances where classical physics fails and only a quantum theory can explain the facts. In squeezed optical field, the fluctuations in one quadrature component are smaller than those associated with a coherent state and it has potential application in reduction of

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noise level in optical communication [8–10] and in detection of the extremely weak gravitational radiation [11–13]. Hong and Mandel [14, 15] generalized the concept of ordinary squeezing to the $2N$-th-order case and Hillery [16, 17] proposed the so-called amplitude-squared squeezing (ASS) which is generated in some nonlinear interactions such as second harmonic generation [16], degenerate parametric amplification [17], two-photon absorption [17], anharmonic oscillator [18], Jaynes–Cummings model [19] etc. The generalization of ASS was carried to orders > 2 and one finds it in literatures [20].

Recently, Prakash and Mishra [21] studied ASS (which can be detected in principle by homodyning method [22]) in a light beam having a Gaussian statistics and reported that this non-classical feature can be enhanced in a simple linear mixing with a classical light beam via a beam splitter. Parametric amplification [23, 24] is one such process, which generates such Gaussian light beams that exhibit non-classical features. We investigate here the possibility of enhancement of amplitude-squared squeezing for Mach–Zehnder interferometer as a next order generalization. It is found that, under some conditions, amplitude-squared squeezing at the output of second beam splitter is enhanced as compared to that at one of output of the first beam splitter. But the maximum amount of amplitude--squared squeezing obtained by the present mixing is not greater than that obtained by Prakash and Mishra [21].

2. Amplitude-squared squeezing in input Gaussian light beam and output beams in Mach–Zehnder interferometer

If we consider hermitian operators $\hat{Y}_{1,2}$ defined by $\hat{Y}_1 + \mathrm{i}\hat{Y}_2 = \hat{a}^2$, i.e., $\hat{Y}_1 = \frac{1}{2}(\hat{a}^{\dagger 2} + \hat{a}^2)$ and $\hat{Y}_2 = \frac{\mathrm{i}}{2}(\hat{a}^{\dagger 2} - \hat{a}^2)$, the non-classical feature called ASS [16, 17] is defined by $\left\langle (\Delta\hat{Y}_1)^2 \right\rangle < \left\langle \hat{a}^\dagger\hat{a} + \frac{1}{2} \right\rangle$ or $\left\langle (\Delta\hat{Y}_2)^2 \right\rangle < \left\langle \hat{a}^\dagger\hat{a} + \frac{1}{2} \right\rangle$. We can also consider the more general operator, $\hat{Y}_\theta \equiv \frac{1}{2}(\hat{a}^{\dagger 2}\mathrm{e}^{\mathrm{i}\theta} + \hat{a}^2\mathrm{e}^{-\mathrm{i}\theta})$ and then ASS (the non-classical feature) occur if

$$Y_\theta \equiv \mathrm{Tr}[\hat{\rho}(\hat{Y}_\theta - \left\langle \hat{Y}_\theta \right\rangle)^2] - [\mathrm{Tr}(\hat{\rho}\hat{a}^\dagger\hat{a}) + \tfrac{1}{2}] < 0. \tag{2}$$

Minimum value of Y_θ can easily be obtained and the non-classical feature can be written as [21]:

$$Y_{\theta,\min} = -\tfrac{1}{2}\left|\left\langle \hat{a}^4 \right\rangle - \left\langle \hat{a}^2 \right\rangle^2\right| + \tfrac{1}{2}[\left\langle \hat{a}^{\dagger 2}\hat{a}^2 \right\rangle - \left\langle \hat{a}^{\dagger 2} \right\rangle \left\langle \hat{a}^2 \right\rangle] < 0. \tag{3}$$

For studying this non-classical feature it is customary [18, 21] to consider the normalized quantity $W_\theta \equiv Y_\theta/[\mathrm{Tr}(\hat{\rho}\hat{a}^\dagger\hat{a}) + \frac{1}{2}]$ too.

Normally-ordered characteristic function of a light beam having a Gaussian statistics may be written as

$$\chi_{\mathrm{N}}(\zeta) = \mathrm{Tr}[\hat{\rho}\exp(\zeta\hat{a}^\dagger)\exp(-\zeta^*\hat{a})] = \exp(-(A\zeta_{\mathrm{r}}^2 + B\zeta_{\mathrm{i}}^2)), \tag{4}$$

where $\zeta = \zeta_{\mathrm{r}} + \mathrm{i}\zeta_{\mathrm{i}}$. The condition that $\chi(\zeta) = \chi_{\mathrm{N}}(\zeta)\exp(-\frac{1}{2}|\zeta|^2) \to 0$ as $\zeta \to \infty$ [25] gives $(A + \frac{1}{2}) > 0$, $(B + \frac{1}{2}) > 0$. Also uncertainty relation $\langle(\Delta\hat{X}_1)^2\rangle\langle(\Delta\hat{X}_2)^2\rangle \geq \frac{1}{4}$ gives $(A + B + 2AB) \geq 0$. Special cases $\theta = 0$ and

$\pi/2$ give $Y_\theta = Y_1 = \frac{1}{2}(A^2 + B^2)$ and $Y_\theta = Y_2 = AB$, respectively. Y_2 can be amplitude-squared squeezed if one of A and B are negative.

Using

$$\Gamma^{(m,n)} = \mathrm{Tr}(\hat{\rho}\hat{a}^{\dagger m}\hat{a}^n) = (-1)^n \partial_\zeta^m \partial_{\zeta^*}^n \chi_\mathrm{N}(\zeta)|_{\zeta=0}, \tag{5}$$

we obtain

$$Y_\theta = \tfrac{1}{2}(A - B)^2 \cos\theta + AB, \tag{6}$$

and

$$Y_\theta/[\mathrm{Tr}(\hat{\rho}\hat{a}^\dagger \hat{a}) + \tfrac{1}{2}] = [(A - B)^2 \cos\theta + 2AB]/(A + B + 1). \tag{7}$$

Now let us consider the mixing of this Gaussian non-classical light beam with a coherent light beam using a Mach–Zehnder interferometer (see Fig. 1). In this scheme, we mix Gaussian light beam having annihilation operator $\hat{a}$ with a coherent light beam having annihilation operator $\hat{b}$ via beam splitter BS1, the annihilation operators $\hat{c}$ and $\hat{d}$ are output of BS1 which then is mixed via a second beam splitter BS2 with a relative phase shift $\mathrm{e}^{\mathrm{i}\delta}$.

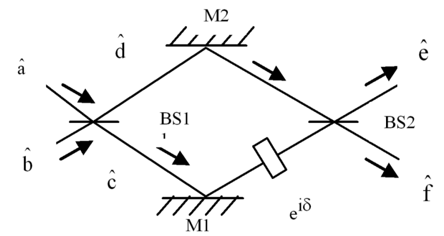

Fig. 1. Mach-Zehnder interferometer.

The different outputs can be written in terms of inputs $\hat{a}$ and $\hat{b}$ [21, 26, 27] as

$$\begin{bmatrix} \hat{c} \\ \hat{d} \end{bmatrix} = \begin{bmatrix} \sqrt{T_1}\exp(\mathrm{i}\phi_{1\mathrm{T}}) & \sqrt{R_1}\exp(\mathrm{i}\phi_{1\mathrm{R}}) \\ -\sqrt{R_1}\exp(-\mathrm{i}\phi_{1\mathrm{R}}) & \sqrt{T_1}\exp(-\mathrm{i}\phi_{1\mathrm{T}}) \end{bmatrix} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}, \tag{8}$$

and

$$\begin{bmatrix} \hat{e} \\ \hat{f} \end{bmatrix} = \begin{bmatrix} \sqrt{T_2}\exp(\mathrm{i}\phi_{2\mathrm{T}}) & \sqrt{R_2}\exp(\mathrm{i}\phi_{2\mathrm{R}}) \\ -\sqrt{R_2}\exp(-\mathrm{i}\phi_{2\mathrm{R}}) & \sqrt{T_2}\exp(-\mathrm{i}\phi_{2\mathrm{T}}) \end{bmatrix} \begin{bmatrix} \hat{c}\mathrm{e}^{\mathrm{i}(\phi_0+\delta)} \\ \hat{d}\mathrm{e}^{\mathrm{i}\phi_0} \end{bmatrix}, \tag{9}$$

where $\sqrt{R_j}\exp(-\mathrm{i}\phi_{j\mathrm{R}})$ and $\sqrt{T_j}\exp(-\mathrm{i}\phi_{j\mathrm{T}})$ $(j = 1,\ 2)$ are the coefficients for reflection and transmission for amplitudes for beam splitter BS1 or BS2, reflection coefficients for mirrors M1 or M2 are equal to $\exp(\mathrm{i}\phi_0)$, and beam reflected by M1 is made to experience a phase shift δ.

Let us consider mixing of a Gaussian light beam (annihilation operator $\hat{a}$) exhibiting ASS with the coherent light beam (annihilation operator $\hat{b}$) of intensity as shown in Fig. 1 and straightforward calculations for minimum value of Y_θ for the a-, c- and e-modes give

$$Y_{\theta a,\min} = AB,\ \mathrm{Tr}(\hat\rho\hat a^\dagger\hat a) + \tfrac{1}{2} = \tfrac{1}{2}(A+B+1), \tag{10}$$

$$\begin{aligned} Y_{\theta c,\min} &= \tfrac{1}{2}T_1^2(A+B)^2 + 2R_1T_1(A+B)|\alpha_0|^2 - \tfrac{1}{2}|T_1^2(B-A)^2\mathrm{e}^{4\mathrm{i}\phi_{1\mathrm{T}}} \\ &\quad +2R_1T_1(B-A)\alpha_0^2\exp(2\mathrm{i}(\phi_{1\mathrm{T}}+\phi_{1\mathrm{R}}))]|, \end{aligned} \tag{11}$$

$$\mathrm{Tr}(\hat\rho\hat c^\dagger\hat c) + \tfrac{1}{2} = R_1^2|\alpha_0|^2 + \tfrac{1}{2}T_1^2(A+B) + \tfrac{1}{2}, \tag{12}$$

$$\begin{aligned} Y_{\theta e,\min} &= \tfrac{1}{4}(\sqrt{T_1T_2}-\sqrt{R_1R_2})^4(A+B)^2 + (\sqrt{T_1T_2}-\sqrt{R_1R_2})^2 \\ &\quad\times(\sqrt{R_1T_2}+\sqrt{T_1R_2})^2(A+B)|\alpha_0|^2 - \tfrac{1}{4}|[T_1T_2\exp(2\mathrm{i}(\alpha_1+\alpha_2+\zeta+\delta)) \\ &\quad +R_1R_2\exp(2\mathrm{i}(\beta_2-\beta_1+\delta)) - 2\sqrt{R_1R_2T_1T_2}\exp(\mathrm{i}(\alpha_1+\alpha_2+\beta_2 \\ &\quad -\beta_1+\delta+2\zeta))]^2(B-A)^2 + 2[T_1T_2\exp(2\mathrm{i}(\alpha_1+\alpha_2+\zeta+\delta)) \\ &\quad +R_1R_2\exp(2\mathrm{i}(\beta_1-\beta_2+\zeta)) + 2[T_1T_2\exp(2\mathrm{i}(\alpha_1+\alpha_2+\zeta+\delta)) \\ &\quad +R_1R_2\exp(2\mathrm{i}(\beta_2-\beta_1+\zeta)) - 2\sqrt{R_1R_2T_1T_2}\exp(\mathrm{i}(\alpha_1+\alpha_2-\beta_1+\beta_2 \\ &\quad +\delta+2\zeta))][R_1T_2\exp(2\mathrm{i}(\beta_1+\alpha_2+\zeta+\delta)) + R_2T_1\exp(2\mathrm{i}(\beta_2-\alpha_1+\zeta)) \\ &\quad +2\sqrt{R_1R_2T_1T_2}\exp(\mathrm{i}(\beta_1+\beta_2-\alpha_1+\alpha_2+\delta+2\zeta))](B-A)|\alpha_0|^2|, \end{aligned} \tag{13}$$

$$\begin{aligned} \mathrm{Tr}(\hat\rho\hat e^\dagger\hat e) + \tfrac{1}{2} &= (\sqrt{R_1T_2}-\sqrt{R_2T_1})^2|\alpha_0|^2 \\ &\quad +\tfrac{1}{2}(\sqrt{T_1T_2}-\sqrt{R_1R_2})(A+B) + \tfrac{1}{2}. \end{aligned} \tag{14}$$

As the ports "*e*" and "*f*" are symmetrical in nature, therefore, we will study the non-classical effect for port "*e*" and a similar discussion may occur for port "*f*".

3. Discussion of the results

With $\phi_{1\mathrm{T}}+\phi_{2\mathrm{T}}+\delta+\zeta = \phi_{2\mathrm{R}}-\phi_{1\mathrm{R}}+\zeta = \gamma/2$, $\sigma_1 \equiv (\sqrt{R_2T_1}-\sqrt{R_1T_2})^2$, and $\sigma_2 \equiv (\sqrt{T_2T_1}-\sqrt{R_1R_2})^2$ we can discuss two cases: (i) for $\alpha_0 = |\alpha_0|\exp(\mathrm{i}(\alpha_1-\beta_1))$ and $(B-A)>0$, $W_{\theta e,\min} = (2\sigma_2^2AB + 4\sigma_1\sigma_2A|\alpha_0|^2)/[\sigma_2(A+B)+2\sigma_1|\alpha_0|^2+1]$; and (ii) for $\alpha_0 = |\alpha_0|\exp(\mathrm{i}(\alpha_1-\beta_1+\pi/2))$, $(A-B)>0$, $W_{\theta e,\min} = (2\sigma_2^2AB + 4\sigma_1\sigma_2B|\alpha_0|^2/[\sigma_2(A+B)+2\sigma_1|\alpha_0|^2+1]$. Taking into account these two restrictions on A and B and on α_0, we can see that $W_{\theta a,\min} < W_{\theta e,\min}$ under some conditions that are dependent on the values of T_j, R_j, $|\alpha_0|^2$, A, and B. Similar conditions may occur in the case of port "*f*". But, it is remarkable to note that in *no way we find the situations where the maximum amount of amplitude-squared squeezing obtained by the present mixing is greater than that reported by Prakash and Mishra* [21]. For example, if we maximize (i.e., most negative), W_a, W_c, and W_e by a simple C++ programming, we find that (i) $T_1 = 0.884992$, $A = -0.1$, $B = 0.125$, $|\alpha_0|^2 = 25$, $W_a = -0.02439$, $W_c = -0.153177$; (ii) $T_1 = 0.9$, $T_2 = 0.6$, $A = -0.1$, $B = 0.125$, $|\alpha_0|^2 = 25$, $W_a = -0.02439$, $W_c = -0.152802$, $W_e = -0.152776$. It is to be noted that Prakash and Mishra reported another non-classical features that can be enhanced by using such a procedure [28, 29].

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