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Holographic Interferometer as a Correlator of Phase Distortions with Response in the Form of Interference Pattern

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Considered in the work are known properties of holographic interferometer as according to the correlator with output signal in the form of interference fringes. It is known that a space correlation has various ratios of intensity between central correlation peak and background part for objects with different phase microrelief of a surface. Correspondingly an interference pattern must have different space distribution of contrast. A magnitude of contrast depends on changes in distribution of microrelief of investigated object, whereas the interference pattern depends on macroscopic modification of a form of the object. Used here is a scheme of Fourier holography with a random phase modulator in signal beam, reference beam is a point source, and restoring beam is the random phase modulator with random phase shift against initial random phase modulator. Also investigated is a recording of holographic interferogram with these random phase modulators. As the results of original theoretical consideration there was obtained an expression for fringe contrast and intensity of correlation peak. This model was tested by simulating of forming of holographic interferogram and reconstruction of the cross-correlation peak with phase distortions of various statistical distributions. Compared here are theoretical results, simulation results, and results obtained in real experiment.

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1. Introduction

In the most general view interferometric experiment can be treated as comparison of two distributions of a field with the purpose of definition of their differ-

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ences during research. At the same time a macroscopic characteristics of a field which describe the form of researched object and microscopic (statistical) characteristics of a field which describe a condition of a surface of object can change. During research and measurements of the form of object the microrelief of a surface can change distorting the results of interferometric researches. But, on the other hand, interference fringes are a remarkable reference point which allows to define sites of object with possible various changes of a microrelief of a surface.

Thus, the problem is reduced to correlation comparison of two fields in which there is a determined and statistical component. The decision of such problem is obtaining of some spatial correlation (rather cross-correlation) functions which simultaneously would determine changes of both the form of object and its surface.

In this work we suggest to consider properties of holographic interferometer as such that correspond to correlator of complex spatial distributions of two fields that have statistical and determined components.

2. Theoretical preconditions

Theoretical consideration, numerical simulation and experimental researches were carried out according to the classical scheme of recording Fourier hologram, but at its correlation reconstructing (by signal beam). The scheme of recording and restoration of the Fourier hologram is reduced to equivalent telescopic system [2], which is represented in Fig. 1.

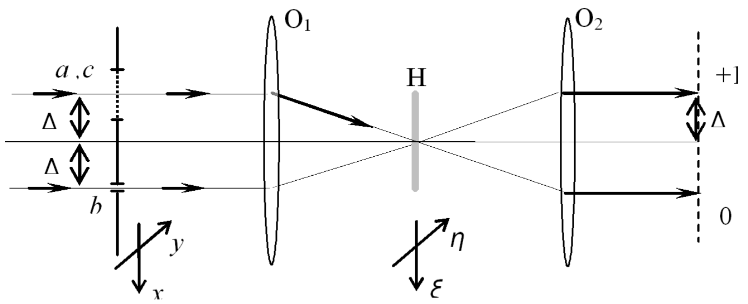

Fig. 1. The generalized scheme of recording and restoration of Fourier hologram. a, c — initial diffuser and diffuser with predefined random distribution phase shift correspondingly, b — point source, O_1, O_2 — Fourier objectives, H — hologram, Δ — shift from the optical axis, x, y and ξ, η — coordinates in front focal plane and in back focal plane of the objective correspondingly, +1 and 0 — diffraction order.

In front focal plane of ((x, y) coordinates) objective O_1 there is located a diffuser and the point source b. In back focal plane of the objective (coordinate (ξ, η)) there is located hologram H. In a subject beam there is placed a diffuser with a randomly distributed phase. The reference beam is the point source; the restoring beam is another diffuser with random phase incursion distribution again the first diffuser.

The mathematics of the operation of the scheme is the following. The complex amplitude of the wave restored in +1 diffraction order in the plane just after the hologram is described as [3]:

$$r_{+1} = F^{-1}\{A^* BC\} = (\tilde{a} * \tilde{c}) \otimes \grave{a}_2, \tag{1}$$

it determines spatial cross-correlation function of the two used diffusers $\tilde{a}$ and $\tilde{n}$ with intersection S. $\tilde{a} = a_1 \exp(\mathrm{i}d(x + \delta y))$, $\tilde{b} = a_2 \delta(x - \Delta, y)$, $\tilde{c} = a_3 \exp(\mathrm{i}d + \psi)$, where ψ — random phase distortion in comparision with diffuser $\tilde{a}$.

Let us write down (1) in an obvious form

$$r_{+1}(0) = a_1 a_2 a_3 \iint_S \exp\left(\mathrm{i}(d - d - \psi)\right) \mathrm{d}x\mathrm{d}y = a_1 a_2 a_3 \iint_S \exp(-\mathrm{i}\psi)\mathrm{d}x\mathrm{d}y \tag{2}$$

and consider the value of the integral

$$\begin{aligned} \iint_S \exp(-\mathrm{i}\psi)\mathrm{d}x\mathrm{d}y &= \iint_S [\cos(\psi) - \mathrm{i}\sin(\psi)]\mathrm{d}x\mathrm{d}y \\ &= \iint_S [\cos(\psi)]\mathrm{d}x\mathrm{d}y - \iint_S [\mathrm{i}\sin(\psi)]\mathrm{d}x\mathrm{d}y. \end{aligned} \tag{3}$$

Let us analyze all over again the second term in the formula (3). The sin function is odd and its argument is a random variable with zero average value. In limits of integral the argument gets values above and below zero in identical quantity. As a consequence, $\iint_S [\mathrm{i}\sin(\psi)]\mathrm{d}x\mathrm{d}y = 0$.

Let us pay attention to the first term in the formula (3). If we divide it by the area of diffuser S it will determine an average value of function $\cos(\psi)$ [4]. On the other hand, average value of $\cos(\psi)$ is defined as: $\int \cos(\psi)\rho(\psi)\mathrm{d}\psi = M[\cos(\psi)]$ [4], where $M[\cos(\psi)]$ is an average value of $\cos(\psi)$, and $\rho(\psi)$ — distribution density of phase incursion.

In the work [5], we studied influence of non-stationary phase distortions in transmitting part of holographic interferometer to the contrast of the interference pattern. Under non-stationary conditions contrast of the interference pattern will be proportional to expression: $V \sim \int \cos(\psi)\rho(\psi)\mathrm{d}\psi$, where ψ — parameter which describes spatial distribution of a phase of random distortions, and $\rho(\psi)$ — distribution density. Considered in the work there were the two most common distributions: Gauss distribution and uniform distribution, also there were obtained the functions of contrast of the interference pattern for these distributions

$$\text{Gauss distributions:} \qquad V' = \exp\left(-\frac{\sigma^2}{2}\right), \tag{4}$$

$$\text{uniform distributions:} \qquad V' = \mathrm{sinc}(a). \tag{5}$$

It is clear that both expressions are determined by the average value of cosine function, as well as in expression (3). From this one may draw a conclusion that the amplitude ($E \sim \iint_S [\cos(\psi)] \mathrm{d}x\mathrm{d}y$ of correlation peak for two diffusers with random phase distortion can be proportional function of contrast of the interference pattern under condition of random phase distortion in comparable objects.

In Fig. 2 there are shown interference patterns for different values σ and a, obtained from simulation.

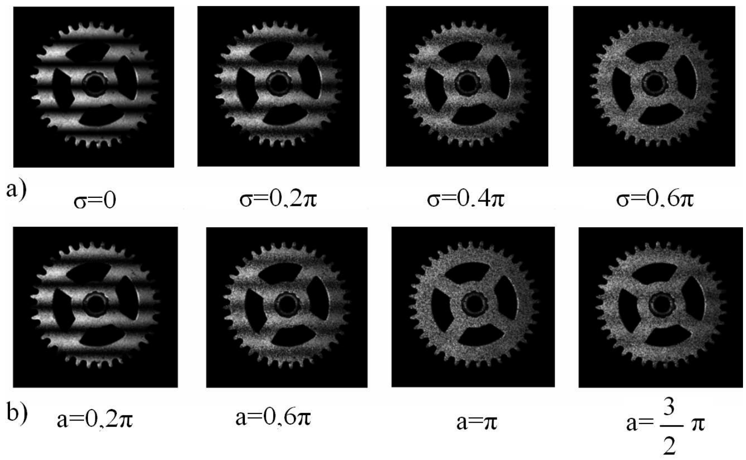

Fig. 2. Interferograms obtained for different values σ and a.

We have ($V' \sim \int \cos(\psi)\rho(\psi)\mathrm{d}\psi$), that is $E \sim V'$. And as $I = |E|^2$, there is clear the fact that

$$I \sim V'^2. \tag{6}$$

3. Experimental results

In Fig. 3 there is represented optical scheme of experiment.

Process of obtaining of experimental results consists of the following stages:

Calculation. The initial diffuser is calculated on a computer in program MatLab. Distributions of a phase in all the next diffusers are compositions of distributions of initial diffuser and random phase distortions with predefined distribution.

Experiment. In Fig. 3 the intensity of correlation peak between initial diffuser and diffusers with a controllable dispersion was registered on optical-digital correlator. For this on SLM (LC-Rr500, reflective, grayscale, resolution 1024×768, bit-depth-8 bit) distribution of a phase of initial diffuser was produced. Angular spectrum of diffuser on hologram H was recorded. Hologram is photopolymer (FPK-488), with diffraction efficiency 55%. Reference to object beam ratio is 10:1. Further on SLM there were consistently produced diffusers with adjusted dispersion. After every new reconstructing diffuser an intensity of correlation peak between diffuser on SLM and initial diffuser recorded on hologram was fixed.

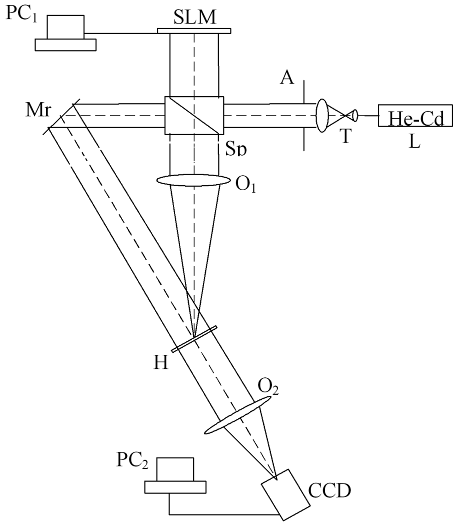

Fig. 3. The scheme of optical-digital correlator. L — He–Cd laser (λ = 441.6 nm), T — telescope, A — aperture, Sp — beam spliter, Mr — revolving mirror, H — hologram, O_1 and O_2 — Fourier objective, SLM — spatial light modulator, PC_1 and PC_2 — computer, CCD — camera.

4. Processing results

In program MatLab there was plot the dependence of intensity correlation peak, received on experiment, either against the dispersion (for Gauss distribution), or against half-width of distribution function for phase distortion (for uniform distribution) of corresponding diffuser. The received dependence was compared to function of contrast of interference fringes for corresponding distributions (4), (5). The result is presented in Fig. 4.

In Fig. 4b there is represented the dependence of intensity of correlation peak for diffusers calculated with uniform distribution and corresponding function of contrast V (5). And in Fig. 4a there is represented the dependence of intensity of correlation peak for diffusers calculated with Gauss distribution, and corresponding function of contrast V (4).

For comparison in Fig. 4 there are shown the results of the simulation intended for some kind of graduation of real experiment. Stages of simulation experiment were analogous to the real one. But the difference was that the second stage was conducted with the help of the PC. For that the results of a theoretical part of this work were used.

5. Conclusion

In this work we have shown that spatial distribution of a field in interference pattern can be treated as convolution of a clean fringe pattern with correlation

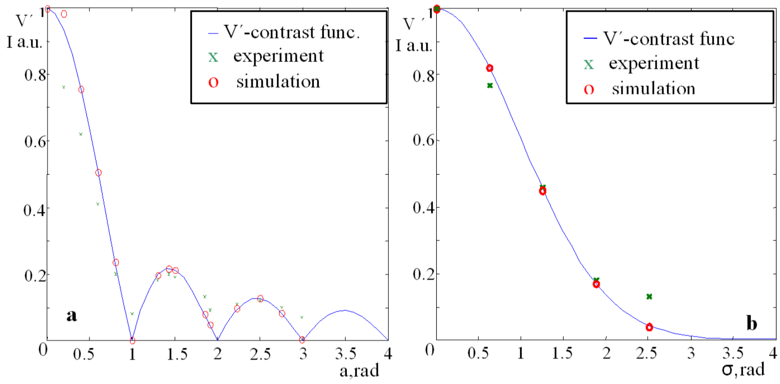

Fig. 4. The dependence of interference pattern contrast V' and of correlation peak intensity I on the scale of phase noise for (a) uniform noise (a — the width of the distribution function) and for (b) Gaussian noise (σ — the dispersion of the distribution).

peak of spatial statistical distributions for phase distortions on a surface of the object. Accordingly the interferometr itself is treated as correlator of statistical fields with a signal in the form of the spatial cross-correlation function, which describes change of the form of the object. Such results expand a scope of holographic interferometry and allow to investigate simultaneously both macro and microscopic characteristics of the object.

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